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$N(4, 4; 3) \geqslant 13$

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This note concerns the Ramsey number $N(4, 4; 3)$, defined as the least n such that for every binary partition $A \cup B$ of the $\binom{n}{3}$ unordered triples in an n-element set, A or B must contain all four triples in some quadruple. It will be shown that $N(4, 4; 3) > 12$. The fact that $N(4, 4; 3) > 11$ is in J. G. Kalbfleisch's 1966 doctoral thesis at the University of Waterloo. $N(4, 4; 3) \leqslant 19$ by a straightforward application of Greenwood and Gleason's evaluation $N(4, 4; 2) = 18$, *Canad. J. Math.* **7** (1955), 1–7 (given by A. Sobczyk, *ibid.* **20** (1968), 520–534).

Consider a set $S = T \times Z_4$, where T is a 3-element set. The 220 triples in S form 48 orbits under the action of the symmetric group $\sum$ on the set T of first coordinates. We index these orbits by 12 letters and 4 subscripts, $A_0, \ldots, L_3$. Putting $T = \{x, y, z\}$, we list representatives of $A_0, B_0, \ldots, L_0$; subscripts shift by translation of Z_4. In A_0 is $T \times \{0\}$; in $B_0\{(x, 0), (y, 0), (x, 1)\}$. Omitting needless punctuation, in C_0 is $(x0)(y0)(z1)$, in D_0 $(x0)(y0)(x2)$, in E_0 $(x0)(y0)(z2)$, in F_0 $(x0)(x1)(y1)$, in G_0 $(x0)(y1)(z1)$, in H_0 $(x0)(x1)(x2)$, in I_0 $(x0)(x1)(y2)$, in J_0 $(x0)(y1)(x2)$, in K_0 $(x0)(y1)(y2)$, in L_0 $(x0)(y1)(z2)$.

One member of the partition is the union of the orbits

$$A_0, A_2, C_0, C_1, C_2, C_3, D_2, D_3, E_1, E_3, F_0, F_1, F_2, F_3, H_1, H_2, I_0, I_2, J_1, J_2, K_0, K_2, L_1, \text{ and } L_3.$$

Of course the verification is not printed here. The 495 quadruples in S fall into 98 orbits under $\sum \times Z_4$.